A dynamic model of CBDC holdings and transactional use*

Nicola Branzoli Tullia Padellini
Elia Moracci Stefano Pietrosanti

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Abstract

We develop a dynamic model of liquidity management and payment choices to study how the design of a central bank digital currency (CBDC) shapes its transactional demand by consumers. Calibrating the model to Italian payment diary data, we study the endogenous substitution between cash, cards, and CBDC by transaction size. We document how a CBDC could achieve a high share of transactions among adopters under realistic design choices and how it could decrease transaction management costs. Whether users can draw from a liquidity source when making CBDC payments emerges as a key design margin. Without this possibility, CBDC payments rely on pre-loaded offline balances, the CBDC is primarily used to settle small purchases and may crowd-in card usage. With it, the CBDC coexists with cash and cards at the counter, achieving greater welfare gains.

Keywords: central bank digital currency, cash management, payment choice
JEL Classification: E41, E42, D12

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1 Introduction

In the euro area, the fraction of cash relative to the total number of point-of-sale (POS) payments decreased from 79 percent in 2016 to 52 percent in 2024, with several forms of digital payments making up the difference. Even more strikingly, the fraction of day-to-day payments occurring online, in environments where cash is not available as a payment option, tripled from 7 percent in 2019 to 21 percent in 2024.[1] Similar trends have emerged in many currency areas[2] and could lower consumer welfare, because the strength and availability of a universal outside option in payment services limits intermediaries' market power (Lagos and Zhang, 2022).

To address such developments, many central banks are considering the introduction of central bank digital currencies (CBDCs).[3] A CBDC would be the digital form of public money, allowing consumers to use it in all situations, both offline and online. Nonetheless, introducing a CBDC could displace deposits and pose sudden disintermediation risks to the banking system (e.g. Ahnert et al., 2023; Bidder, Jackson, and Rottner, 2024). On the other hand, a CBDC could displace cash, with potentially negative effects on resilience (Malaika, Soderberg, and Zhabska, 2025). To avoid disruptions while preserving consumer choices, central banks considering CBDCs stress their commitment to the coexistence of cash, cards, and CBDCs.[4] Thus, CBDC planning requires fine-tuning of features to ensure a balanced mix of payment options for consumers.

This complex policy design problem has attracted considerable academic interest, with special attention being dedicated to how future CBDC holdings may substitute cash and deposit holdings (Li, 2023; Lambert et al., 2024) and to the macroeconomic and monetary policy implications under different design choices (Fernández-Villaverde et al., 2021; Andolfatto, 2021; Brunnermeier and Niepelt, 2019; Chiu et al., 2023; Keister and Sanches, 2023; Niepelt, 2024; Abad, Nuño, and Thomas, 2025). Comparatively less attention has been dedicated to the mi-

[1] See Study on the payment attitudes of consumers in the euro area (SPACE).

[2] In many such areas the degree of digitalization is even higher than in Europe. For example, the most recent figure for the fraction of cash transactions relative to the total number of transactions in the US is 14 percent (O'Brien and Phelps, 2026).

[3] At the moment, 146 countries and currency unions are exploring CBDCs as a possible digital form for public money. See The Atlantic Council's CBDC tracker for continuously updated figures.

[4] For example, in a March 18th, 2025 interview with the Italian financial newspaper *Il Sole 24 Ore*, Banca d'Italia's Governing Board member and Deputy Governor Chiara Scotti stated that "*[the] digital euro will not replace cash and other existing means of payment, but it will offer an alternative, additional option.*"

croeconomics of the transactional use of CBDC. How, why, and to what extent will CBDC transaction flows substitute other means of payment at the counter? How will adopters manage their CBDC balances? In this paper, we propose a framework to answer these questions jointly.

We apply a dynamic inventory theoretic model of liquidity management (cash vs CBDC) and payment choices (cash, cards and CBDC), built on Lippi and Moracci (2026). The dynamic setting enables us to study both *offline* CBDC holdings and cash holdings (stocks) and their use in transactions (flows).[5] The resulting joint stock and flow approach captures the value of the inherently dynamic CBDC features, such as the capacity of CBDC wallets to pay even when empty, transferring money from a linked bank account.[6]

The model describes the behavior of a representative consumer facing a stream of purchase opportunities that arrive at random and vary in size. The agent chooses (i) which means of payment to employ for each purchase, given her cash and CBDC holdings (a *payment policy*), and (ii) after each transaction, whether to replenish her stocks of cash and CBDC, potentially facing withdrawal costs (a *withdrawal policy*). The model features several realistic frictions: some merchants may not accept cards and/or CBDC, or connectivity may fail, limiting the capacity for digital transactions. Importantly, purchase opportunities can be missed; agents may find themselves in situations where they lack any feasible payment option.

A key strength of this model is that it captures consumers' precautionary behavior at the counter, driving endogenous substitution between CBDC, cash, and card usage. For example, such precautionary behavior implies that consumers will not opt for a cash-burn policy (Alvarez and Lippi, 2017)—i.e., since cash is always accepted, consumers will not always use it when it is possible to do so. Instead, they may settle small transactions with cards to spare cash and insure against the risk of missing future payments, which aligns with observed behavior (see Lippi

[5]We stress "offline" in "*offline* CBDC holdings" because CBDC holdings may exist in two forms. *Online holdings* would be stored in a CBDC account and would be usable at the counter only when point-of-sale terminals are able to communicate with the financial system via an internet connection. The decision to store liquidity this way would be akin to the decision to hold liquid saving buffers (see, e.g. Kaplan and Violante, 2014). *Offline holdings*, by contrast, would be a fraction of these balances and would be stored locally on chips equipped with secure elements (e.g., the same hardware that already hosts payment card information on smartphones). These holdings would be usable without internet access and share key properties with cash; they would be private in use and exposed to the risk of loss if the hosting device is lost. Thus, a dynamic inventory model of payments and liquidity usage is a perfect setting to study them. For more details on offline usage in the European case, see the September 2025 interview with the Digital Euro Program Director, Evelien Witlox, *"ECB's Evelien Witlox on design choices for the digital euro"*, for Central Banking.

[6]What in the context of the Digital Euro is called the *reverse waterfall functionality*.

and Moracci, 2026). Introducing a CBDC provides consumers additional insurance against missed transactions by offering a payment method that is less costly to carry and top up compared to cash. However, CBDC may be slightly more cumbersome to use at the counter, as the user must still interact with an app or a dedicated card. The actual design of the CBDC will determine whether adopters perceive it as a further means of conserving cash—hence a card substitute—or as an alternative to holding cash and settling transactions in an emergency—a cash substitute.

After presenting our theoretical framework and describing consumers' objectives, we solve for the resulting optimal withdrawal and payment policies for consumers in a world in which cash, cards, and CBDC are all available. To perform quantitative experiments, we calibrate the non-CBDC block of the model to Italian micro-moments from the 2023-24 wave of the Study on the Payment Attitudes of Consumers in the Euro Area (SPACE), focusing on physical POS transactions. On this basis, we conduct counterfactuals over parameter vectors that summarize the most salient CBDC design features, including its opportunity cost of holding, the fixed cost of using it at the POS, the share of merchants that accept it, and the availability of a liquidity source.[7] For each scenario, the calibrated model delivers predictions on the average cash and offline CBDC balances, the shares of usage at the POS, the substitution from cash and cards towards CBDC to settle purchases of different sizes, as well as the user reliance on a source of liquidity—either a bank account linked to the CBDC wallet, or online CBDC holdings—to fund CBDC usage.

Our **findings** can be summarized as follows. First, the presence of a liquidity source is a crucial design margin. If users can only pay in CBDC through their offline holdings, consumers hold on to their offline CBDC balances for precautionary reasons, use them mainly for small purchases, and the CBDC mostly substitutes cash, both in their pockets and at the point of sale. With a costlessly accessible liquidity source, offline CBDC holdings are smaller, while usage is more frequent and extends to larger transactions; cash, cards and CBDC usage coexist. Second, different CBDC designs affect the precautionary motive for using cards differently and could crowd-in cash or card usage at the counter. On the one hand, lower cash balances could increase the opportunity cost of using public money at the counter, increasing cards' share; on the other, the availability of another means of payment could lower such opportunity cost, rising cash's share. In our simulations,

[7] We stress that these costs are time/inconvenience costs in monetary equivalents, not actual fees.

an offline-only design could crowd-in card transactions.

Given these results, we analyze the welfare effects of introducing a CBDC, focusing on how such a choice impacts the cost of managing transactions for adopters. In the model, this cost equals the sum of the opportunity cost of holding cash and offline CBDC balances, the fixed costs of cash withdrawals and CBDC top-ups, the usage costs of cards and CBDC, and the utility cost of missed purchases. The CBDC reduces these costs by providing an additional payment rail; however, the size and composition of the gain depend sharply on the design. With a liquidity source, the CBDC provides a two-layered insurance against the risk of missing purchases, through the connection with a liquidity source whenever internet is available and through always available offline balances. Without it, consumers obtain such insurance only by holding larger offline CBDC buffers, which limits the welfare gain.

These results highlight why a joint model of payment stocks and payment flows is necessary for policy evaluation. CBDC holdings or transaction shares, studied in isolation, are insufficient to assess the consequences of CBDC design: the same CBDC transaction share may correspond to very different effects on cash demand and changes in the roles played by cards. At the same time, our quantitative exercises should be interpreted as short-run counterfactuals for adopters. Indeed, for now, we abstract from the endogenous adoption decision, from heterogeneity across consumers, and from medium-run responses by merchants, banks, and card networks. Within these limits, the model provides a disciplined laboratory for evaluating how design features such as acceptance, usage costs, and the availability of links to different sources of liquidity shape the coexistence of cash, cards, and CBDC at the point of sale, by modifying the opportunity cost of using each means of payment.

Related literature and contribution. This paper is related to two strands of literature. First, it builds on inventory models of the demand for cash, a strand of the literature initiated by Tobin (1956); Baumol (1952) and expanded by Frenkel and Jovanovic (1980); Whitesell (1989); Alvarez and Lippi (2009); Alvarez and Lippi (2013); Alvarez and Lippi (2017). This literature regards public money as a means of payment of last resort, held by consumers in the same way that shops may hold empty space on their shelves to avoid missing opportunities to stock useful goods. The model we propose is an augmented version of the cash management and payment choices model by Lippi and Moracci (2026). We contribute by demonstrating that the prudential driver of payment choices highlighted in the latter paper plays a key role in determining substitution patterns when a new

means of payment enters the market. Will the new option be a means of payment of last resort, like cash, or a source of insurance against missing *future* payments, like cards? Different CBDC designs will imply sharply different answers to the above question, as well as substitution patterns at the counter and in the wallet.

In proposing such a contribution, this paper relates to the microeconomic literature on potential CBDC demand. Existing research on this topic has followed two separate approaches to evaluate potential holdings and transactional use. One strand of the literature focuses on usage in transactions, exploiting survey data and BLP structural demand models to conduct counterfactual exercises on the fraction of payments made with each instrument after the introduction of a CBDC (e.g. Huynh et al., 2024; Nocciola and Zamora-Pérez, 2024). Another strand uses similar tools to project CBDC holdings (e.g. Li, 2023; Lambert et al., 2024), estimating the counterfactual conversion from deposits and cash to CBDC.

While such approaches, together with reduced-form empirical work (e.g. Abramova et al., 2022; Bijlsma et al., 2024), are crucial for studying and quantifying the role of users' heterogeneous preferences and characteristics, a joint, dynamic model of means of payment holding and utilization uniquely allows us to study the dynamic interdependency of stock and flow decisions, and how CBDC design affects it. Overall, we try to address the challenge of delivering a "more realistic" model of a CBDC (i.e., closer to the current central bankers' debate), focusing on the immediate aftermath of a CBDC introduction and on its role as an additional means of payment (Bindseil and Senner, 2025).

Roadmap. The present draft is structured as follows. Section 2 introduces a dynamic, inventory-theoretic model of cash management and payment choice, augmented with a CBDC. Section 3 presents our calibration strategy and outline the different combinations of CBDC features that we study in our quantitative exercises. Section 4 discusses the results of model simulations.

2 The model

A consumer finances an expenditure stream driven by a compound Poisson process, whose cumulative value at time t is $e(t) = \sum_{i=1}^{N(t)} s_i$. Here, $\{N(t) : t \geq 0\}$ is a Poisson process with rate λ and $\{s_i : i \geq 1\}$ are i.i.d. random variables with a continuous cumulative distribution function $F(s)$ satisfying $F(0) = 0$, where s denotes the size of purchases.[8] The future is discounted at the rate $\rho > 0$.

[8]Compound-Poisson expenditure processes are standard in continuous-time inventory models of liquid-asset management and payment choice (e.g., Alvarez and Lippi, 2013; Lippi and

Baseline model. In the baseline environment, the consumer has access to two payment methods: cash and a payment card. Let $m \in \mathcal{M} := [0, +\infty)$ denote cash balances. Holding m units of cash entails an opportunity cost Rm, while adjusting cash holdings through a withdrawal involves a fixed cost $b > 0$. Cash balances must be sufficient to cover the transaction in order to settle a purchase with cash, i.e. $m \geq s$. Using the payment card entails a fixed cost κ, independent of the value of the purchase. Cards are accepted in a fraction $\phi \in (0, 1)$ of stores, and for any purchase, there is a probability ι of connectivity failure at the POS; as a result, cards can be used only for a fraction $\phi(1 - \iota)$ of transactions. When cash on hand is insufficient to carry out a purchase and cards cannot be used, the agent cannot consume. We assume that this event entails a utility loss u, independent of the size of the missed purchase. The consumer chooses optimally when to adjust cash holdings and which payment method to use upon the arrival of each purchase opportunity. Let $v(m)$ denote the value function of this baseline problem. The Bellman equation characterizing $v(m)$ is reported in Appendix Section A.

Introducing CBDC. We extend this environment by introducing a central bank digital currency (CBDC). As we focus on data from the euro area, our scenarios and modeling choices are inspired by the debate surrounding the possible introduction of a digital euro. We abstract from the extensive margin of adoption and assume that all agents have access to the option to pay in CBDC; we only focus on the intensive margin of usage. The decision problem becomes more involved, as the consumer now manages two liquid assets. Let d denote the stock of CBDC balances held in the digital wallet and available offline. Holding d units of offline CBDC entails an opportunity cost $R_d d$, with $0 < R_d < R$. To refill CBDC offline balances, agents pay a fixed cost b_d, with $0 \leq b_d < b$. The fixed cost (or benefit, if negative) of paying using the CBDC compared to paying with cash is denoted by κ_d. As the model is in partial equilibrium, an exogenous share ϕ_d of stores accepts CBDC payments.

There are two major differences between CBDC and cards. First, if the stock of CBDC available offline is sufficient to carry out a purchase ($s \leq d$), the CBDC can be used even if there is no connectivity; in this respect, it is cash-like. Second, if connectivity does not fail at the time of payment, agents can pay using CBDC even when $s > d$, either exploiting further online-only CBDC balances or the linkage between the CBDC wallet to a bank account. In this case, the extra

Moracci, 2026). The substantive simplifying restriction is a constant, memoryless arrival rate, which allows us to treat current balances as sufficient state variables.

CBDC balances needed to settle the purchase are drawn by paying a fixed cost $\widetilde{b}_d$. The presence of such a liquidity source makes the CBDC more card-like, especially when $\widetilde{b}_d$ is negligible. Even when they have access to a CBDC, consumers can still miss a purchase opportunity and sustain a utility loss u. This occurs either when $s <= d$ and $s > m$ but the store does not accept both cards and CBDC, or when (i) $s > d$ and $s > m$; (ii) cards cannot be used either because connectivity failed or the store does not accept cards; and (iii) the bank account link or additional online CBDC holdings cannot be used either because the internet connection is unavailable or the store does not accept CBDC payments.[9]

While further online CBDC holdings could be a source of additional liquidity to accomplish POS purchases, we stress that we cannot study them in this setting. Indeed, such holdings would be a form of liquid saving in a consumption and saving problem (e.g., Aiyagari, 1995; Kaplan and Violante, 2014). To include them in the picture, we either need a model encompassing store of value motives to hold liquid savings as online CBDC holdings or deposits, or survey data to pinpoint total desired holdings. In the rest of the sections before the conclusion, for simplicity in exposition, we will assume that the liquidity source is a linked bank account. Because our exercises are inspired to the policy debate around the introduction of a Digital Euro, we will call this link a *reverse waterfall functionality - RWF* and will refer to the two scenarios we compare as "with RWF"/"without RWF".

Let $v(m, d)$ denote the value function for an agent holding m units of cash and d units of CBDC. Let $m^*(d) = \arg\min_{\hat{m}} v(\hat{m}, d)$ denote the optimal level of cash holdings conditional on having d units of CBDC, and let $v^*_m(d) = v(m^*(d), d)$. Similarly, let $d^*(m) = \arg\min_{\hat{d}} v(m, \hat{d})$ denote the optimal level of CBDC holdings conditional on having m units of cash, and let $v^*_d(m) = v(m, d^*(m))$. Finally, let $(m^{**}, d^{**}) = \arg\min_{\hat{m},\hat{d}} v(\hat{m}, \hat{d})$, and let $v^{**} = v(m^{**}, d^{**})$ denote the value associated with jointly adjusting both assets.

The introduction of a CBDC, together with heterogeneous payment acceptance across merchants and the possibility of internet outages, generates five possible situations that can occur when the consumer faces a payment opportunity, sum-

[9]Considering the digital euro as the CBDC of reference, modeling non-acceptance should not be interpreted as questioning the envisaged legal tender status of the CBDC. Indeed, the digital euro Draft Regulation foresees acceptance exceptions for micro-enterprises that do not accept electronic means of payment (see section 2 of the Draft Regulation's Explanatory Memorandum). In addition, effective acceptance may differ from *de jure* acceptance because merchants could steer customers informally towards alternative payment methods. Given the short-run, partial equilibrium nature of our framework, treating widespread but incomplete acceptance as the baseline scenario aims at capturing transitional implementation frictions rather than the long-run legal tender regime.

Table 1: Payment availability combinations

Case	Probability	Cash	Card	CBDC	RWF
I	$(1-\phi_d)(1-\phi(1-\iota))$	✓			
II	$\phi_d\iota$	✓		✓	
III	$(1-\phi)\phi_d(1-\iota)$	✓		✓	✓
IV	$\phi(1-\phi_d)(1-\iota)$	✓	✓		
V	$\phi\phi_d(1-\iota)$	✓	✓	✓	✓

Note: Payment methods available to the consumer at a payment opportunity, depending on merchant acceptance (ϕ, ϕ_d) and internet availability ($1-\iota$). Cash is always available. The reverse waterfall functionality (RWF) requires both CBDC acceptance and internet connectivity.

marized in Table 1. In Case I, the store does not accept either cards or CBDC, or the internet is down and the store does not accept CBDC: the consumer can only pay with cash. In Case II, the store accepts CBDC but the internet is down, so the RWF is unavailable and the consumer chooses between cash and CBDC holdings. In Case III, the store accepts CBDC and the internet is available but cards are not accepted: the consumer can use cash or CBDC, and the RWF can be exploited to settle purchases exceeding the CBDC balance. Case IV is the mirror image: the store accepts cards but not CBDC, so the consumer chooses between cash and cards as in the pre-CBDC world. Finally, in Case V all three instruments are available and the RWF is operative, giving the consumer the widest set of options.

In all five cases, cash can be used only to finance purchases of size $s \leq m$. Similarly, without RWF, the CBDC can only be used to finance purchases of size $s \leq d$. The value function satisfies the Bellman equation

$$v(m,d) = \min\left\{ \underbrace{v_m^*(d)+b}_{\text{adjust only cash}} \;,\; \underbrace{v_d^*(m)+b_d}_{\text{adjust only CBDC}} \;,\; \underbrace{v^{**}+b+b_d}_{\text{adjust both wallets}} \;,\; \underbrace{\frac{Rm+R_d d+\lambda\widetilde{v}(m,d)}{\rho+\lambda}}_{\text{adjust neither and wait}} \right\},$$

where $\widetilde{v}(m,d)$ denotes the expected continuation value upon encountering a payment opportunity, which includes all the possible cases I-V reported above. As its complete expression is quite involved, we report $\widetilde{v}(m,d)$ in Section B.1.

Model solution. The model solution is described by a value function $v(m,d)$ that solves the above Bellman equation, two withdrawal policy functions $a_m(m,d)$ and $a_d(m,d)$ (one for cash and one for CBDC balances) and two payment choice policy functions $p_1(m,d,s)$ and $p_2(m,d,s)$. For any level of current cash and CBDC balances (m,d), a_m and a_d yield the optimal amount of cash to be with-

drawn or the optimal amount of CBDC to be added to the digital wallet, respectively. Define the optimal (discrete) adjustment choice

$$\begin{aligned}\mathcal{A}(m,d) = \text{argmin}_{a\in\{1,2,3,4\}} &\mathbb{1}(a=1)\cdot[v_m^*(d)+b]+\\ &+\mathbb{1}(a=2)\cdot[v_d^*(m)+b_d]+\\ &+\mathbb{1}(a=3)\cdot[v^{**}+b+b_d]+\\ &+\mathbb{1}(a=4)\cdot\left(\frac{Rm+R_d d+\lambda\widetilde{v}(m,d)}{\rho+\lambda}\right).\end{aligned}$$

Then, optimal adjustments of cash a_m and CBDC a_d are given by

$$a_m(m,d) = \begin{cases} 0 & \text{if} \quad \mathcal{A}(m,d)=2 \vee \mathcal{A}(m,d)=4,\\ m^*(d)-m & \text{if} \quad \mathcal{A}(m,d)=1,\\ m^{**}-m & \text{if} \quad \mathcal{A}(m,d)=3,\end{cases}$$

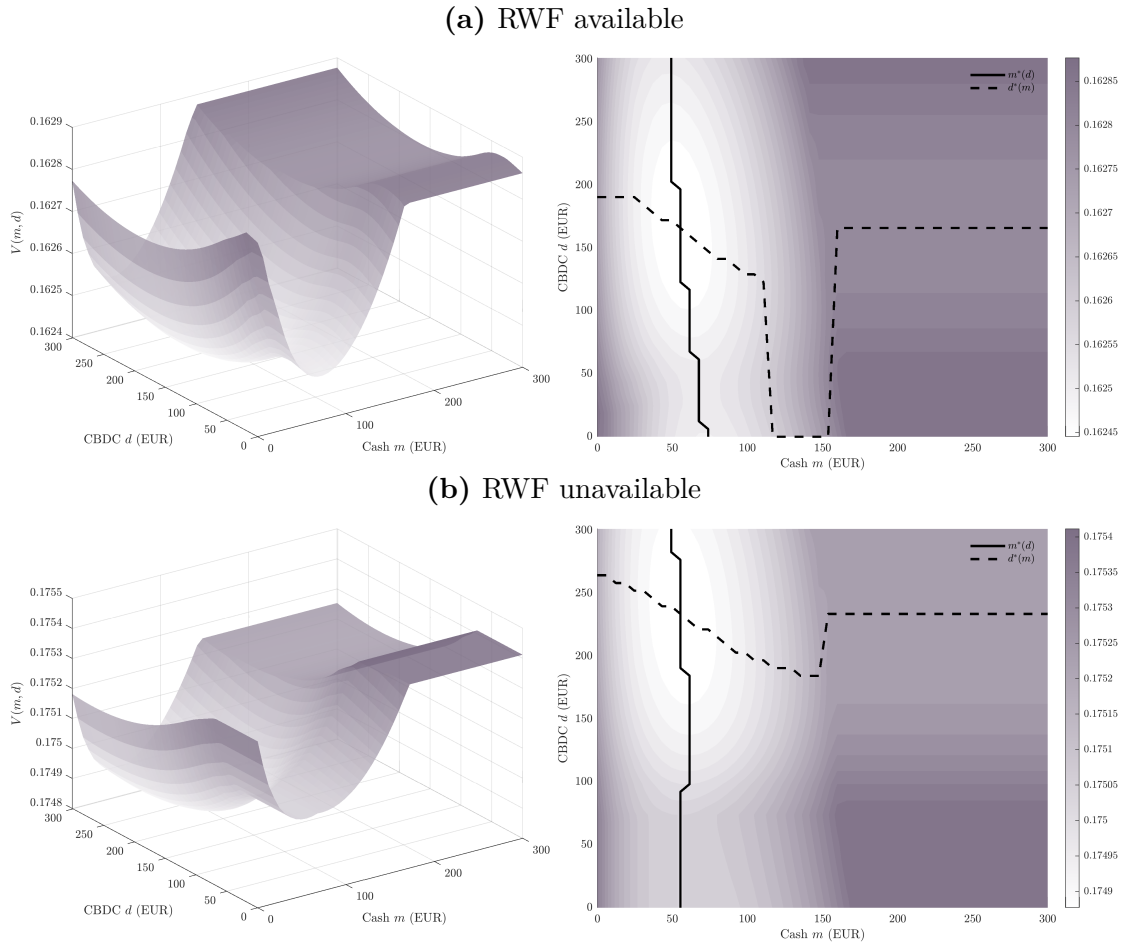
and

$$a_d(m,d) = \begin{cases} 0 & \text{if} \quad \mathcal{A}(m,d)=1 \vee \mathcal{A}(m,d)=4,\\ d^*(m)-d & \text{if} \quad \mathcal{A}(m,d)=2,\\ d^{**}-d & \text{if} \quad \mathcal{A}(m,d)=3.\end{cases}$$

Similarly, for any level of current cash holdings m, CBDC balances d, and the size of the incoming transaction s, let $p_1(m,d,s), p_2(m,d,s) \in \{1,2,3\} = \{\text{cash}, \text{card}, \text{CBDC}\}$ yield the optimal and second best payment option, respectively. Notice that the second best payment option is a relevant object, as the first best choice could be unavailable to the agent, who should (possibly) choose between the two remaining alternatives. Also, notice that $p_1(m,d,s) \neq 1$ and $p_2(m,d,s) \neq 1$ whenever $s > m$, as cash is not a feasible option in such cases. Instead, it may be that $p_1(m,d,s) = 3$ or $p_2(m,d,s) = 3$ even when $s > d$, as CBDC can still be used when balances are smaller than the size of the transaction through the RWF (when available).

Model-implied moments. Given policy functions $\{a_m, a_d, p_1, p_2\}$, it is possible to obtain model-implied statistics. As closed-form expressions cannot be derived, we resort to simulations. We track the payment choices and cash and CBDC management decisions of a cross-section of individuals over time and then compute sample averages. We are interested in the following list of statistics. First, we want to compute the average amount of cash/CBDC in consumers physical/digital

Figure 1: Value function $v(m, d)$ and optimal holdings.



Note: The Figure displays the value function $v(m, d)$ for our baseline calibration (see Section 3 for details). Left panels: 3-D surface. Right panels: contour plot with optimal conditional holdings $m^*(d)$ (solid) and $d^*(m)$ (dashed). Panel (a): RWF available; panel (b): RWF unavailable.

wallets, over time, rescaled by the mean total daily expenditures e.[10] These two moments are denoted by M/e (normalized cash holdings) and D/e (normalized CBDC holdings). The (marginal) stationary distributions for cash and CBDC holdings $h(m)$ and $h(d)$ are of interest as well. Second, we are interested in evaluating the share of expenditure (purchases) settled using each of the three payment methods. Let γ and γ_d be the proportion of *expenditure* settled using cards and CBDC, with the residual $1-\gamma-\gamma_d$ of expenditure being settled in cash. Similarly, let $\widetilde{\gamma}_n$ and $\widetilde{\gamma}_{n,d}$ be the share of *purchases* settled using cards and CBDC.

[10]Importantly, we assume that consumption levels (summarized by e in our model) are unchanged after the introduction of a CBDC, as our model abstracts from potential macroeconomic effects of CBDC that affect household consumption.

Features of the solution. Figure 1 displays the value function $v(m, d)$, illustrating how the cost of managing transactions changes given initial stocks of cash (m) and offline CBDC (d) under our baseline calibration.[11] A few interesting features of the model are already apparent from visual inspection. First, we observe a complex interplay between m and d. As the contour plot in panel (a) shows, when the RWF is available, the optimal level of cash holdings $m^*(d)$ (solid line) depends on the amount of CBDC on hand d, revealing some form of substitutability: for low levels of d, the consumer optimally holds more cash to compensate, and vice versa. The same applies for $d^*(m)$ (dashed line). Second, comparing the 3-D surfaces, the value function is always lower in panel (a) than in panel (b), reflecting the benefit of a liquidity source in terms of lower risk of missing transactions and diminished needs to bear liquidity management costs.[12] Third, adjustment policies vary depending on the availability of the RWF: in panel (b), the contour plot shows that $d^*(m)$ is less elastic to m, indicating that CBDC and cash holdings are managed more independently when the CBDC is not linked to an online liquidity source. Finally, we note that the biggest difference with respect to a setting without CBDC is that, in such a model, it is always optimal to refill cash balances whenever they fall below a certain positive threshold $\underline{m}$. Introducing a CBDC weakens the precautionary motive to visit ATMs when cash balances are still positive and makes such decision dependant on the amount of offline CBDC available to the user.[13]

3 Calibration and CBDC design scenarios

In this part of the document, we discuss our calibration. Section 3.1 describes our calibration strategy for the non-CBDC block of the model, based on the ECB SPACE 2024 data on consumers' payment habits and preferences in Italy. Section 3.2 describes our assumptions about the parameters of the CBDC block, discussing how we use different parameter configurations to mimic alternative CBDC design features.

[11] We present the calibration in Section 3. The associated adjustment policy function $\mathcal{A}(m, d)$ is displayed in Figure B1 in the Appendix.

[12] The welfare analysis in Section 4 will support and deepen this conclusion.

[13] For comparison, the value function $v(m)$ for a model without CBDC but with otherwise identical parameterization is displayed in Figure A1 in the Appendix. The above conclusion is also apparent from Figure B2 in the Appendix, where we also display the stationary distributions for cash and CBDC holdings (h_m and h_d respectively) under our baseline calibration.

Table 2: Non-CBDC block: calibration and model fit.

Calibrated parameters		**Model fit**		
Parameter	Value	Moment	Data	Model
Externally calibrated		*Targeted moments*		
Purch. size, location μ_s	-1.07	λ (purch. freq., yearly)	747	761
Purch. size, scale σ_s	0.99	M/e (cash / daily exp.)	1.18	1.22
Purch. size, shape ν_s	0.23	n^* (yearly withdrawals)	94.5	95.1
Discount rate ρ	0.05	γ (card share, value)	0.55	0.57
Card acceptance rate ϕ	0.89	$\widetilde{\gamma}$ (unforced, value)	0.45	0.44
Internet outage prob. ι	0.025	*Non-targeted moments*		
Opp. cost, cash R	0.1	M (mean cash, €)	71.0	72.5
Internally calibrated		M/c (cash / cash exp.)	2.61	2.88
Withdrawal cost b	0.022	W/M (withdr. / cash)	1.49	1.33
Card usage cost κ	0.009	γ (card share, number)	0.41	0.25
Purch. frequency λ	773.1	Forced cash (value)	0.09	0.09
Missed purch. cost u	0.28	Forced card (value)	0.25	0.32

Note: Left panel: calibrated parameters for the non-CBDC block. Right panel: model fit to Italian payment diaries from SPACE 2024.

3.1 Calibration strategy for the non-CBDC block

For the non-CBDC block, we calibrate the model following the strategy of Lippi and Moracci (2026) (LM from now on) with a few minor adjustments. The parameters of the non-CBDC version of the model are $\boldsymbol{\Theta} = \{\rho, \phi, F, R, b, \kappa, \lambda, u, \iota\}$. All such parameters are calibrated using the Italian sample of the 2024 SPACE survey, employing the same strategy utilized by LM whenever possible (i.e., except for ι, which we add). We briefly outline the approach and refer to the discussion in LM for details.

Externally calibrated parameters. The annual discount rate is such that $\rho = 0.05$, so $\beta = 1/(1+\rho) \approx 0.95$. The opportunity cost of holding cash is set to $R = 10\%$ in annual terms.[14] The card-acceptance rate ϕ is directly related to the share of Italian stores that do not accept cards in SPACE 2024. The parameter ι is set to 0.025 as an upper bound to the risk of technical difficulties at the counter.We slightly depart from LM in our choice of the transaction size distribution F. To achieve a better fit to the actual distribution of purchase sizes, we use a more flexible generalized Gamma specification instead of a lognormal one. We estimate the expenditure size distribution parameters (μ_s, σ_s, ν_s) by maximum likelihood and then shift its location μ_s to normalize the (intended) overall yearly expenditure

[14]This value is an upper-bound on cash opportunity costs found in the empirical literature and survey studies, e.g. Knümann, Krüger, and Seitz (2025).

to 365. Figure C1 in the Appendix displays how our calibrated transaction size distribution compares with the actual distribution of purchase sizes in the data.

Internally calibrated parameters. The remaining parameters are $\{b, \kappa, \lambda, u\}$, which are chosen via a minimum distance procedure that targets five moments computed on the Italian sample of SPACE 2024, namely (i) the ratio between average cash balances and daily expenditures, (ii) the number of cash withdrawals per year, (iii) the card share of expenditure γ, (iv) the card share of unforced expenditure $\widetilde{\gamma}$, and (v) the number of realized purchases per year $\hat{\lambda}$. Table 2 displays the calibrated parameters and the model fit to Italian payment diaries from SPACE. Note that κ, b, u are expressed in monetary terms; for example, $u = 0.28$ implies that every missed transaction "costs" the representative user 28 EUR cents.

3.2 Alternative CBDC configurations and assumptions

The relevant dimensions for our model in the CBDC block are captured via the parameter vector $\mathbf{\Theta}_d = \{R_d, \kappa_d, b_d, \widetilde{b}_d, \phi_d\}$, which encompasses the following CBDC design features: R_d represents the opportunity cost of holding a CBDC; therefore, $R - R_d$ is the extra inconvenience of carrying cash relative to a CBDC, including the increased risk of balances being lost or stolen and higher storage costs; κ_d represents the fixed cost of using the CBDC instead of cash for a given purchase, including privacy/security concerns, frictions in usability, and merchants' preference for cash; b_d represents the fixed cost associated with a CBDC withdrawal or top-up; $\widetilde{b}_d$ represents the fixed cost of using the RWF, which we assume null whenever the RWF is available; finally, ϕ_d denotes the CBDC acceptance rate by merchants.[15]

Baseline scenario and sensitivity analysis. Table 3 summarizes the CBDC design parameters, distinguishing between the values used in our baseline scenario and the ranges explored in the sensitivity analysis. In the baseline, we set the opportunity cost of holding CBDC to $R_d = 0.01$, the usage cost to $\kappa_d = 0.75\kappa$ (a slight improvement over cards), and the merchant acceptance rate to $\phi_d = 0.92$ (slightly above the current card acceptance in Italy). For each experiment, we additionally compare two polar RWF regimes: $\widetilde{b}_d = 0$ (costless RWF) and RWF

[15] CBDC-related costs are time/inconvenience costs in monetary terms, not actual fees.

Table 3: CBDC block: baseline scenario and sensitivity ranges.

Description	Param.	Baseline	Sensitivity
Opportunity cost, CBDC	R_d	0.01	$[0.001, R]$
CBDC usage cost	κ_d	0.75κ	$[0, \kappa]$
CBDC withdrawal cost	b_d	$0.5b$	—
RWF cost (if available)	$\widetilde{b}_d$	0	—
CBDC acceptance rate	ϕ_d	0.92	[0.86, 0.98]

Note: CBDC design parameters. The *Baseline* column reports the values used in the baseline scenario (Section 4.1). The *Sensitivity* column reports the range explored in the comparative statics (Section 4.2); a dash indicates parameters held fixed throughout. The RWF is additionally toggled between available (with $\widetilde{b}_d = 0$) and unavailable in all experiments.

unavailable.[16]

Concerning the sensitivity ranges, $R_d \in [0.001, R]$ spans the full spectrum from a nearly costless digital public money to one as expensive to hold as physical cash. The grid for κ_d ranges from 0 to κ: this spans the entire spectrum of payment frictions at the point-of-sale, with a fully cash-like ($\kappa_d = 0$) and a fully card-like ($\kappa_d = \kappa$) CBDC as extremes. The acceptance grid $\phi_d \in [0.86, 0.98]$ ranges from medium-high, in line with current card acceptance by Italian merchants, to near-universal.[17] Finally, we set the withdrawal/top-up cost $b_d = 0.5b$, a positive value lower than the cash withdrawal cost b. This assumption stems from the fact that a wallet top-up will still involve some interaction with the app or device through which the consumer accesses the digital euro, even though it will not require visiting an ATM physically.

Caveats. We stress four fundamental caveats before introducing preliminary results. First, all the quantitative results we obtain are conditional on adoption; in the present version of our model, the agent is endowed with the option to hold and use a CBDC and does not face an adoption problem. Hence, for example, simulated shares of CBDC transactions must be read as *shares of CBDC transactions among the adopters*. Second, as briefly discussed above, our framework is a model of the *short-run*, as it assumes that the environment faced by consumers does not change after the CBDC is introduced in any other respect. Hence, while the model

[16] As a baseline assumption, costless and universally available RWF functionalities is coherent with the present draft of recital (40) in the European Commission's Digital Euro draft regulation.

[17] For a study on merchant acceptance as a driver of CBDC diffusion (see Bai et al., 2025). For more details about digital euro design plans, see the latest release of the ECB's digital euro FAQs, which we take as a reference for our assumptions and scenarios throughout this Section.

can be used to perform welfare calculations, these evaluations unavoidably abstract from how competing means of payment will adapt after the introduction of a CBDC (e.g., card networks may lower their fees, making card payments accepted more often by merchants). Third, model-based CBDC balances are meant to represent *offline transactional holdings*, as we do not model consumption-saving motives underpinning deposit holdings and store-of-value CBDC balances. Nonetheless, the availability of a link between the CBDC as a means of payment and either further online CBDC balances or the deposit holdings of the consumer, or both, plays an important role in our setting. Finally, the present version of the model abstracts from heterogeneity, exclusively providing insights into the potential behavior of the average Italian consumer as captured in SPACE 2024.

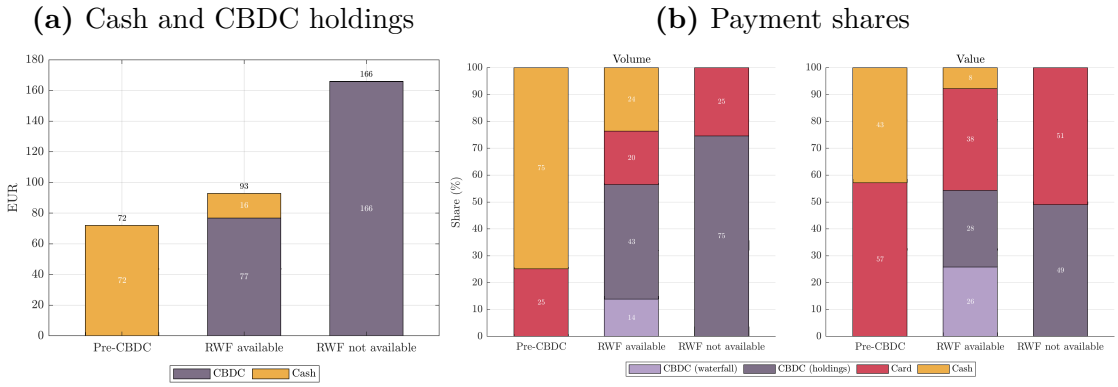
4 Results

Under the parametrizations presented in Tables 2 and 3, we discretize the Bellman equation resulting from the decision problem in Section 2 and solve it using value function iteration. We describe the results under the baseline scenario and discuss comparative statics with respect to CBDC characteristics (R_d, ϕ_d, κ_d), always comparing with and without RWF outcomes. The key takeaway from our analysis is that the availability of the RWF dramatically changes the impact of CBDC issuance from a close substitute for cash in small transactions, which may, in some cases, crowd-in card usage, to a payment option that coexists with cash and cards.

4.1 Baseline scenario

Cash and CBDC holdings. Figure 2(a) displays average cash and CBDC holdings (in EUR) at the baseline calibration. When the RWF is available, total liquidity holdings remain relatively close to pre-CBDC levels (93 EUR vs 72 EUR), with consumers holding a modest cash buffer of approximately 16 EUR. When the CBDC is not linked to an online liquidity source, however, the representative payer's cash holdings drop to zero, and CBDC holdings rise to 166 EUR. In this offline-only counterfactual, the representative adopter substitutes entirely away from transactional cash holdings. Since the opportunity cost of holding CBDC in the baseline scenario is lower than that of cash, consumers hold larger liquid (CBDC) buffers for precautionary reasons. This conditional portfolio result does not imply withdrawal of cash or its disappearance among non-adopters.

Figure 2: Baseline scenario: holdings and payment shares



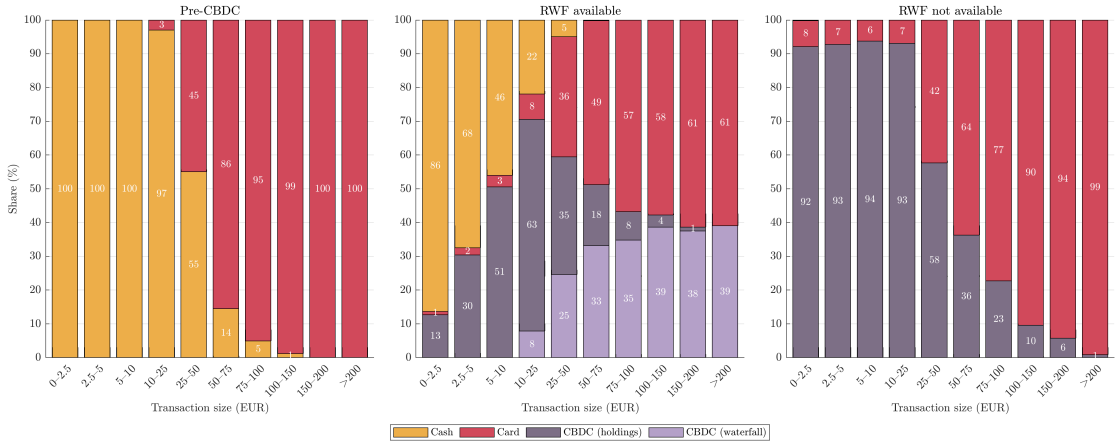
Note: Panel (a): average cash and CBDC holdings (in EUR) in the pre-CBDC world, with RWF available, and without. Panel (b): share of daily payments by instrument, in volume (left) and value (right) terms.

Payment shares. Figure 2(b) shows payment shares at the baseline. Again, the RWF proves critical. In a design where only offline balances can be used to pay, the CBDC captures 75% of transactions by volume at the expense of cash, while card usage remains largely unchanged. With the RWF, CBDC usage is spread more evenly: it accounts for 57% of transactions by volume and 54% by value, partly substituting both cash and card payments both. In this last case, it is important to note that the link to a liquidity source accounts for almost half of the value transacted on the CBDC rail and for one quarter of the volume. Such a fact shows how the RWF allows a CBDC to be competitive in mid-to-large-sized day-to-day transactions. In the model, without the chance to handle substantial transactions seamlessly, a CBDC would mostly be used for small-value purchases, which are typically settled using cash in the pre-CBDC world.

Payment shares by transaction size. We can deepen this last finding by examining the distribution of usage shares by transaction size. In Figure 3 we plot the share of the volume of transactions settled with each instrument across transaction size bins. In the no RWF case (Figure's rightmost panel), CBDC usage concentrates among small transactions (below 50 euros), an area where cash dominated before the introduction of CBDC. As a consequence, the CBDC mostly substitutes cash at the POS. When the RWF is available, a different picture emerges: CBDC is used across the full spectrum of transaction sizes, substituting both cash and cards at the POS.[18]

[18] Figure B3 in the Appendix further decomposes CBDC substitution patterns, showing which pre-existing payment method is replaced for each transaction size bin, further supporting our

Figure 3: Payment shares in volume by transaction size



Note: Model-implied share of transactions settled with each payment instrument (cash, cards, CBDC) across transaction size bins. Transactions are grouped into 10 bins of equal frequency. The first panel from the left provides the payment share by transaction size bin when no CBDC is available, for comparison; the second panel displays shares when the RWF is available; the third panel displays shares when the RWF is not available.

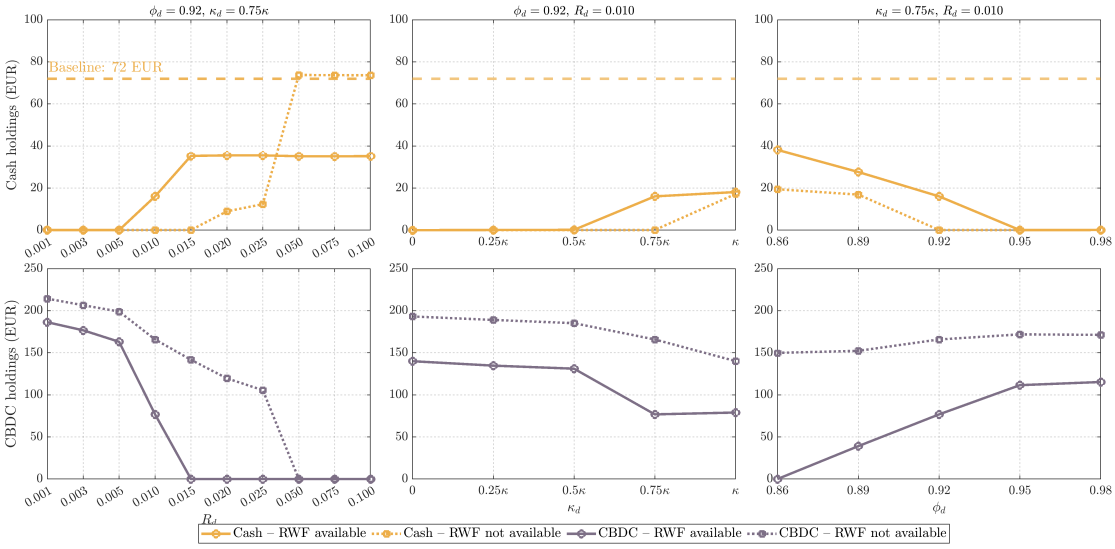
4.2 Sensitivity to other design choices

CBDC holdings across design configurations. The sensitivity of CBDC holdings to design parameters is displayed in Figure 4. Without an online liquidity source, CBDC holdings are large and less sensitive to CBDC features, reflecting how CBDC overtakes the role of cash as the main precautionary buffer against the risk of losing transaction opportunities among users. With the RWF, first, offline holdings are intuitively smaller across the entire parameter space, as the liquidity source provides a backstop; second, offline holdings are more reactive to CBDC features.

CBDC usage across design configurations. We now turn to the comparative statics of expenditure shares with respect to the CBDC design parameters. In Figure 5, we display the share of daily expenditures (in value terms) settled with each payment method after the introduction of CBDC, compared with the corresponding pre-CBDC shares. The RWF is critical for CBDC usage shares as well. Without an online liquidity source, the CBDC transaction share may become economically significant; however, this occurs only in a minority of cases by substituting cards at the counter. When RWF is enabled, this pattern changes substantially: rather than solely competing with cash for small transactions, a

argument.

Figure 4: Cash and CBDC holdings: sensitivity to CBDC design

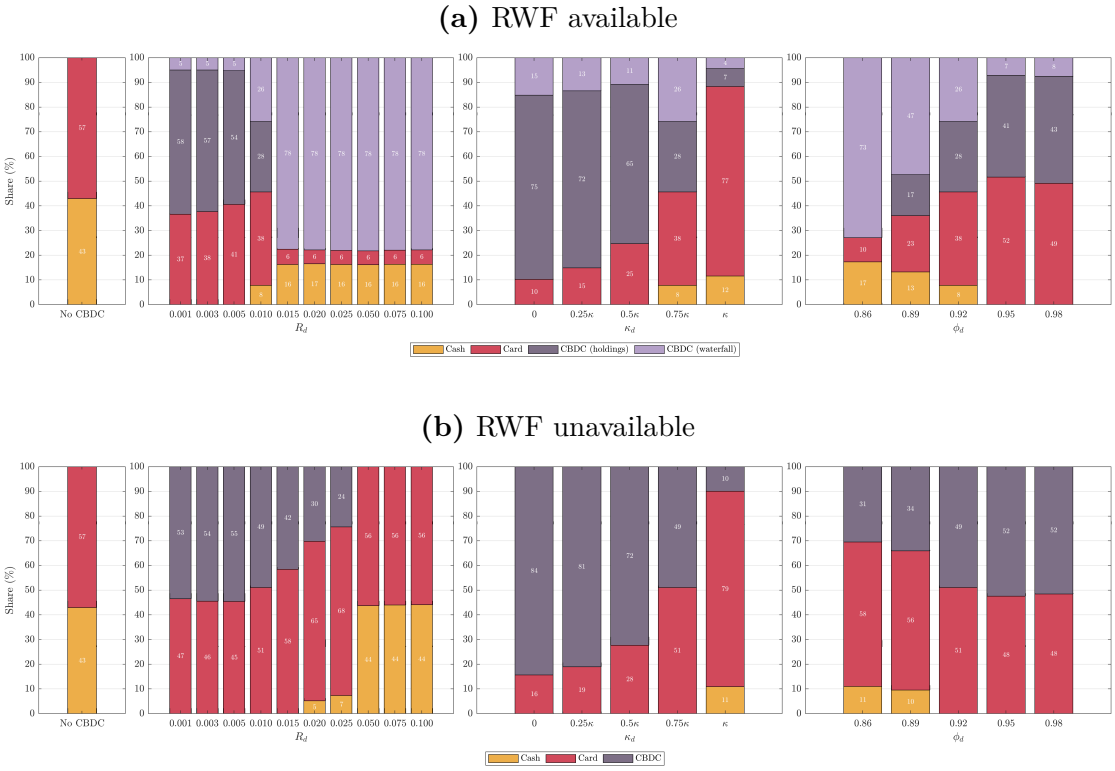


Note: Average cash and offline CBDC holdings (in EUR) as a function of CBDC design parameters. Each panel varies one parameter while holding the others fixed at the baseline values.

CBDC linked to a liquidity source competes across the full transaction spectrum, affecting both cash and card payment shares (Figure 5, panel (a)).

Welfare value of CBDC design features. Figure 6 summarizes the model-implied cost of managing transactions across three environments: the pre-CBDC baseline, a CBDC with a costless RWF, and one with no RWF. Each stacked bar decomposes the annual cost borne by the representative consumer into the opportunity cost of cash holdings (yellow), the opportunity cost of CBDC holdings (purple), card usage costs (red), CBDC usage costs (light violet), cash withdrawal costs (light gray), CBDC top-up costs (dark gray), and the utility cost of lost purchases (dark blue). The figure highlights two key messages. First, payment-related welfare gains from introducing a CBDC stem from reducing costly liquidity management and better insuring consumers against missed transactions. Second, when an RWF is available, the welfare gains also reflect lower card usage and fewer lost purchases, while CBDC balances and top-up costs remain modest because the linked source of liquidity provides payment insurance at the point of sale. In the design lacking the RWF, welfare gains stem almost completely from the substitution of cash usage and holding, leaving consumers with higher liquidity-management costs than under the RWF design.

Figure 5: Share of payment value by instrument: sensitivity to CBDC design



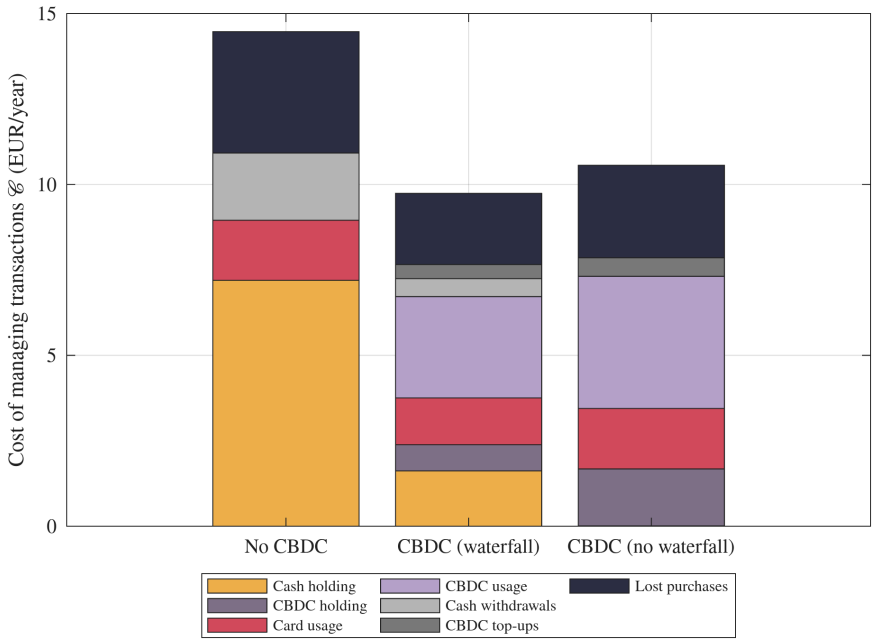
Note: Share of daily payments in value, by instrument and CBDC design parameter. Each panel varies one parameter while holding the others fixed at the baseline values.

State-contingent crowding-in of payment methods. Another key result we document is the existence of parameter regions for which the new means of payment generates a *crowding-in* of the pre-existing ones. In our example, some CBDC design configurations result in a *higher* card share of expenditures among CBDC users. As we can see from Figure 5, the crowding-in of cards is most common when users can only pay with CBDC through offline balances and the opportunity cost of holding CBDC is relatively high.[19]

Crowding-in of card usage at the counter stems from adjustments in the *stock* of cash held by consumers. The introduction of a CBDC in our simulations weakly decreases the stock of cash held by CBDC users (see Figure 4). If the CBDC is not linked to an online liquidity source, such a drop in the cash stock is more

[19]For very high opportunity costs of holding, the usage of a CBDC that can pay only through offline balances collapses. Indeed, usage is tied to offline holdings in such scenarios, and a CBDC that is cumbersome to hold would see users rarely replenishing their wallets. We consider such cases quite unrealistic, as the opportunity cost of holding CBDC is credibly a fraction of that of cash. For example, the amount stored in a digital wallet can be easily verified when the wallet is online, making it possible to supply cheap insurance services concerning stolen devices.

Figure 6: Cost of managing transactions by CBDC design

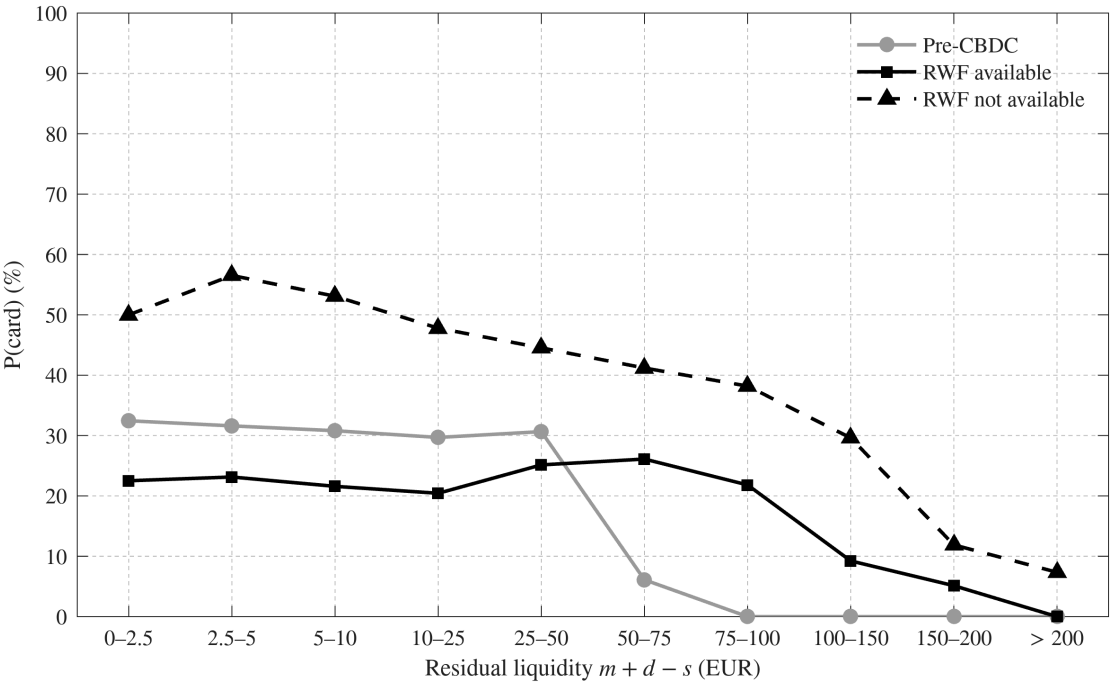


Note: Annual model-implied cost of managing transactions, decomposed by source. The no-CBDC bar reports the baseline model with cash and cards only; the other bars report the corresponding costs after CBDC introduction with and without access to a liquidity source.

dramatic, as offline holdings and usage are tied, and adopters may opt to almost fully substitute cash with its less cumbersome digital equivalent. In some cases, the resulting representative users' offline holdings of public money may be relatively low, and the value of insuring transaction opportunities through the use of cards may rise. Instead, introducing a CBDC with a direct link to an online liquidity source increases the number of payment rails available and weakens the insurance role of cards. The crowding-in of cards at the counter is the prevalent crowding-in scenario, given the internally and externally calibrated parameters. Nonetheless, the opposite force is active in the model as well. Indeed, introducing a CBDC decreases, *ceteris paribus*, the risk of missing transactions. If such a decrease in risk were to be quantitatively predominant, people may end up using cash more freely for transactions that they would have otherwise settled using cards.

To visualize such changes in the prudential payment behavior at the counter, in Figure 7 we plot the probability of using cards as a function of implied residual liquidity $l' = m + d - s$ in the baseline scenario. We define residual liquidity as the sum of the CBDC (d) and cash (m) on hand after an incoming payment of size s is settled using part of the initial liquidity stock. For clarity of display in the plot, we focus on *non-forced* transactions—transactions that could have been

Figure 7: Change in prudential behavior of payers



Note: Model-implied probability of paying with cards as a function of the implied residual amount of liquidity $l' = m + d - s$ if the transaction would have been settled in cash or CBDC.

settled with liquidity if the user wished to do so. This way, we avoid accumulation points where only cards are available as an option, and their usage probability is forced to 100%.

In the Figure, we display the pre-CBDC usage probability with a solid light shaded line (in this case, residual liquidity is $l' = m' = m - s$), the CBDC with the RWF option's usage probability with a solid dark shaded line, and the CBDC without the RWF option's usage probability with a dashed dark shaded line. Pre-CBDC patterns display a steep relationship between residual liquidity and the probability of card usage around the 50 EUR bracket; above that amount, the probability of card usage is almost null, while below that, it is about 30%, because users wish to preserve cash. When we introduce a CBDC linked to a liquidity source, the line depicting the probability of card usage flattens, decreasing for small residual liquidity amounts while increasing for larger ones, highlighting the diminishing importance of prudential card usage. Finally, in the scenario of an offline-only CBDC, the conditional probability of using cards jumps, dominating the pre-CBDC line for all types of non-forced transactions, due to the greater resulting demand for missed transaction insurance.

5 Conclusion

This paper develops a dynamic inventory-theoretical framework to study CBDC transactional holdings and usage jointly, with explicit payment method choices across cash, cards, and CBDC. The model addresses the current policy debate around the potential role of CBDCs in the payments market by incorporating realistic design margins (CBDC holding costs, payment frictions, acceptance, and the links to sources of liquidity) while preserving a disciplined microfoundation for payment behavior.

Our study highlights three core messages. First, the presence of a direct link to a liquidity source is a first-order driver of usage: without it, a CBDC may gain an economically significant market share mainly by substituting cash for small transactions; with it, a CBDC can become relevant across the full transaction-size distribution, coexisting with cash and cards. Second, depending on design and merchant acceptance, introducing a CBDC can crowd-in card or cash payments due to dynamic self-insurance decisions. Under the present model calibration, our simulations suggest that a CBDC design requiring offline holdings to pay may crowd-in card transactions. Third, introducing a CBDC increases users' welfare by reducing missed transactions and decreasing liquidity management costs. Design choices making CBDC usage less dependent on holdings, such as bank account linkages, increase welfare gains.

These findings highlight how policymakers should focus on the specific way a CBDC — or any new means of payment, more generally — enters the payments market. Does it gain shares in the settlement of small transactions, large transactions, or both? Do consumers use the new means of payment through holdings downloaded onto a wallet, or do they use it as a new way to mobilize their liquidity at the counter? For each means of payment, the same aggregate market share can mask different implications for transactional cash demand, card usage, and welfare.

References

Abad, Jorge, Galo Nuño, and Carlos Thomas (2025). "CBDC and the operational framework of monetary policy". In: *Journal of Monetary Economics* 151, p. 103762.

Abramova, Svetlana et al. (2022). *What can CBDC designers learn from asking potential users? Results from a survey of Austrian residents.* Working Papers 241. https://www.econstor.eu/handle/10419/264833. Oesterreichische Nationalbank.

Ahnert, Toni et al. (2023). *Central bank digital currency and financial stability.* Working Paper 2783. ECB. URL: %7Bhttps://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2783~0af3ad7576.en.pdf%7D.

Aiyagari, S. Rao (1995). "Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting". In: *Journal of Political Economy* 103.6, pp. 1158–1175.

Alvarez, Fernando and Francesco Lippi (2009). "Financial innovation and the transactions demand for cash". In: *Econometrica* 77.2, pp. 363–402.

— (2013). "The demand of liquid assets with uncertain lumpy expenditures". In: *Journal of Monetary Economics* 60.7, pp. 753–770.

— (2017). "Cash burns: An inventory model with a cash-credit choice". In: *Journal of Monetary Economics* 90, pp. 99–112.

Andolfatto, David (2021). "Assessing the impact of central bank digital currency on private banks". In: *The Economic Journal* 131.634, pp. 525–540.

Bai, HaiChen et al. (2025). "Adoption of central bank digital currencies: Initial evidence from China". In: *Journal of Corporate Finance*, p. 102735.

Baumol, William J. (1952). "The transactions demand for cash: An inventory theoretic approach". In: *The Quarterly Journal of Economics* 66.4, pp. 545–556.

Bidder, Rhys, Timothy P Jackson, and Matthias Rottner (2024). *CBDC and banks: Disintermediating fast and slow.* Discussion Paper 15. https://www.bundesbank.de/resource/blob/931090/be2be8b2c5324245e4147d6306689312/mL/2024-04-29-dkp-15-data.pdf. Deutsche Bundesbank.

Bijlsma, Michiel et al. (2024). "What triggers consumer adoption of central bank digital currency?" In: *Journal of Financial Services Research* 65.1, pp. 1–40.

Bindseil, Ulrich and Richard Senner (2025). "Modeling central bank digital currencies". In: *Journal of Economic Surveys* 39.5, pp. 2088–2105.

Brunnermeier, Markus K and Dirk Niepelt (2019). "On the equivalence of private and public money". In: *Journal of Monetary Economics* 106, pp. 27–41.

Chiu, Jonathan et al. (2023). "Bank market power and central bank digital currency: Theory and quantitative assessment". In: *Journal of Political Economy* 131.5, pp. 1213–1248.

Fernández-Villaverde, Jesús et al. (2021). "Central bank digital currency: Central banking for all?" In: *Review of Economic Dynamics* 41, pp. 225–242.

Frenkel, Jacob A. and Boyan Jovanovic (1980). "On transactions and precautionary demand for money". In: *The Quarterly Journal of Economics* 95.1, pp. 25–43.

Huynh, Kim P et al. (2024). "Demand for Payment Services and Consumer Welfare: The Introduction of a Central Bank Digital Currency". In: *Review of Network Economics* 23.4, pp. 199–230.

Kaplan, Greg and Giovanni L. Violante (2014). "A model of the consumption response to fiscal stimulus payments". In: *Econometrica* 82.4, pp. 1199–1239.

Keister, Todd and Daniel Sanches (2023). "Should central banks issue digital currency?" In: *The Review of Economic Studies* 90.1, pp. 404–431.

Knümann, Fabio, Malte Krüger, and Franz Seitz (2025). *Costs of Cash and Card Payments from a Consumer Perspective*. Study by Deutsche Bundesbank. https://www.bundesbank.de/resource/blob/954346/5b205843c942907eba04ff6b91662593/472B63F073F071307366337C94F8C870/kosten-von-bargeld-kartenzahlungen-verbrauchersicht-2024-data.pdf.

Lagos, Ricardo and Shengxing Zhang (2022). "The limits of ONETARY economics: On money as a constraint on market power". In: *Econometrica* 90.3, pp. 1177–1204.

Lambert, Claudia et al. (2024). *Digital euro demand: design, individuals' payment preferences and socioeconomic factors*. Working Paper 2980. European Central Bank. URL: %7Bhttps://papers.ssrn.com/sol3/papers.cfm?abstract_id=4958524#%7D.

Li, Jiaqi (2023). "Predicting the demand for central bank digital currency: A structural analysis with survey data". In: *Journal of Monetary Economics* 134, pp. 73–85.

Lippi, Francesco and Elia Moracci (2026). "Cash or card? A structural model of payment choices". In: *Journal of Monetary Economics*, p. 103930.

Malaika, Majid, Gabriel Soderberg, and Kateryna Zhabska (2025). *Payment resilience in fragile and conflict-affected states: lessons for central bank digital currency (CBDC)*. IMF Fintech Notes. https://www.imf.org/-/media/files/publications/ftn063/2025/english/ftnea2025009a.pdf.

Niepelt, Dirk (2024). "Money and Banking with Reserves and CBDC". In: *The Journal of Finance* 79.4, pp. 2505–2552.

Nocciola, Luca and Alejandro Zamora-Pérez (2024). *Transactional demand for central bank digital currency*. Working Paper 2926. ECB. URL: %7Bhttps://papers.ssrn.com/sol3/papers.cfm?abstract_id=4797200%7D.

O'Brien, Shaun and Hailey Phelps (2026). *2026 Findings from the Diary of Consumer Payment Choice*. Report. The Federal Reserve Financial Services. URL: %7Bhttps://www.frbservices.org/wp-content/uploads/2026-diary-consumer-payments-choice.pdf%7D.

Tobin, James (1956). "The interest-elasticity of transactions demand for cash". In: *The Review of Economics and Statistics* 38.3, pp. 241–247.

Whitesell, William C. (1989). "The demand for currency versus debitable accounts: note". In: *Journal of Money, Credit and Banking* 21.2, pp. 246–251.

A Baseline model without CBDC

This Appendix describes the baseline environment without a CBDC and reports the HJB equation characterizing the consumer's problem. The framework closely follows Lippi and Moracci (2026).

A consumer finances an exogenous expenditure stream driven by a compound Poisson process. The cumulative value of expenditures up to time t is given by

$$e(t) = \sum_{i=1}^{N(t)} s_i,$$

where $\{N(t) : t \geq 0\}$ is a Poisson process with arrival rate λ and $\{s_i : i \geq 1\}$ are i.i.d. random variables with continuous cumulative distribution function $F(s)$ satisfying $F(0) = 0$. The variable s denotes the size of purchases. The future is discounted at rate $\rho > 0$.

The consumer has access to two payment methods: cash and a payment card. Let $m \in \mathcal{M} := [0, +\infty)$ denote the stock of cash balances. Holding m units of cash entails an opportunity cost Rm. Cash balances can be adjusted through withdrawals from a bank account at a fixed cost $b > 0$. Cash can be used to pay for a purchase only if the available cash stock is sufficient to cover the transaction, i.e. if $m \geq s$.

Using a payment card entails a fixed cost κ, independent of the size of the purchase. Cards are accepted in a fraction $\phi \in (0, 1)$ of stores. In addition, for each purchase there is a probability ι of connectivity failure at the POS, in which case card payments are not feasible. As a result, cards can be used only for a fraction $\widetilde{\phi} = \phi(1 - \iota)$ of transactions.

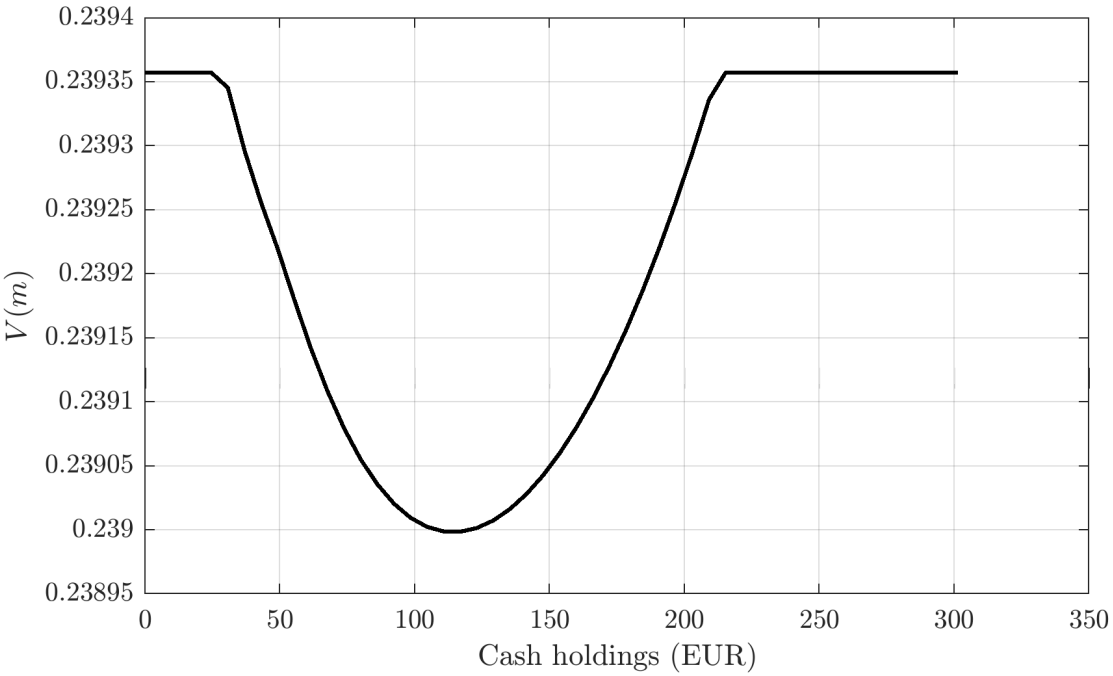
When cash on hand is insufficient to carry out a purchase and cards cannot be used, the consumer is unable to consume. We assume that this event entails a utility loss u, independent of the size of the missed purchase.

Let $v(m)$ denote the value function for an agent holding m units of cash. Define the target level of cash balances as

$$m^* = \arg\min_{\hat{m}} v(\hat{m}),$$

and let $v^* = v(m^*)$ denote the value associated with optimally adjusting cash

Figure A1: Value function $v(m)$ in the model without CBDC.



holdings. The value function satisfies the HJB equation

(1)

$$\rho\, v(m) = \min\Bigg\{\rho\left(v^* + b\right), Rm + \lambda\left(1 - F(m)\right)\left(\widetilde{\phi}\kappa + \left(1 - \widetilde{\phi}\right)u\right) + \lambda\int_0^m \left[\widetilde{\phi}\min\left\{v(m-s) - v(m),\, \kappa\right\} + \left(1 - \widetilde{\phi}\right)\left(v(m-s) - v(m)\right)\right]dF(s)\Bigg\}.$$

The outer minimization captures the consumer's choice between adjusting cash holdings by paying the fixed cost b and resetting cash balances to the optimal level m^*, or not adjusting cash and continuing with the current stock m. In the latter case, with probability $1 - F(m)$ the incoming purchase exceeds available cash balances. If cards can be used, which occurs with probability $\widetilde{\phi}$, the consumer pays the fixed card cost κ and completes the transaction; otherwise, the purchase is lost and the consumer incurs the utility loss u. When the purchase size satisfies $s \leq m$, the consumer can pay with cash. If cards are available, the consumer chooses optimally between paying with cash—reducing the state from m to $m - s$—and paying with a card, incurring the fixed cost κ while keeping cash balances unchanged. When cards are not available, the consumer must use cash and the state transitions to $m - s$.

Figure A1 displays the value function $v(m)$ that solves (1) for our baseline

calibration of the model without CBDC, outlined in Section 3.

B Theoretical model

B.1 Detailed expression for $\widetilde{v}$

Recall the five payment availability cases summarized in Table 1. We now outline the full expression for $\widetilde{v}$, using colors to map each branch to a specific case: Case I, Case II, Case III, Case IV, and Case V. For $m \geq d$, $\widetilde{v}$ is given by

$$
\begin{aligned}
&\text{(I)} \\
&\widetilde{v}(m,d) = (1-\phi_d)\,(1-\phi(1-\iota))\,(1-F(m))\,(v(m,d)+u) \\
&\text{(I)} \qquad + (1-\phi_d)\,(1-\phi(1-\iota)) \int_0^m v(m-s,d)dF(s) \\
&\text{(II)} \qquad + \phi_d\iota\,(1-F(m))\,(v(m,d)+u) \\
&\text{(II)} \qquad + \phi_d\iota \int_d^m v(m-s,d)dF(s) \\
&\text{(II)} \qquad + \phi_d\iota \int_0^d \min\left\{v(m-s,d), v(m,d-s)+\kappa_d\right\} dF(s) \\
&\text{(III)} \qquad + (1-\phi)\phi_d(1-\iota)\,(1-F(m))\left(v(m,0)+\kappa_d+\widetilde{b}_d\right) \\
&\text{(III)} \qquad + (1-\phi)\phi_d(1-\iota) \int_d^m \min\left\{v(m-s,d), v(m,0)+\kappa_d+\widetilde{b}_d\right\} dF(s) \\
&\text{(III)} \qquad + (1-\phi)\phi_d(1-\iota) \int_0^d \min\left\{v(m-s,d), v(m,d-s)+\kappa_d\right\} dF(s) \\
&\text{(IV)} \qquad + \phi(1-\phi_d)(1-\iota)\,(1-F(m))\,(v(m,d)+\kappa) \\
&\text{(IV)} \qquad + \phi(1-\phi_d)(1-\iota) \int_0^m \min\left\{v(m-s,d)\ ,\ v(m,d)+\kappa\right\} dF(s) \\
&\text{(V)} \qquad + \phi\phi_d(1-\iota)\,(1-F(m)) \min\left\{v(m,d)+\kappa, v(m,0)+\kappa_d+\widetilde{b}_d\right\} \\
&\text{(V)} \qquad + \phi\phi_d(1-\iota) \int_d^m \min\left\{v(m-s,d), v(m,d)+\kappa, v(m,0)+\kappa_d+\widetilde{b}_d\right\} dF(s) \\
&\text{(V)} \qquad + \phi\phi_d(1-\iota) \int_0^d \min\left\{v(m-s,d), v(m,d)+\kappa, v(m,d-s)+\kappa_d\right\} dF(s).
\end{aligned}
$$

Notice that the actual continuation value upon encountering a payment opportunity depends three things: (i) on the payment method acceptance policy of the store visited (accept only cash, accept both cash and cards, accept both cash and CBDC, accept all payment methods), (ii) on whether the internet connection is available or not, and (iii) on the size of the purchase relative to the amount of

cash and CBDC held by the consumer. For $m < d$, $\widetilde{v}$ is given by

$$
\begin{aligned}
&\text{(I)} \\
&\widetilde{v}(m,d) = (1-\phi_d)\,(1-\phi(1-\iota))\,[(1-F(m))\,(v(m,d)+u)]+ \\
&\text{(I)} \qquad + (1-\phi_d)\,(1-\phi(1-\iota))\left[\int_0^m v(m-s,d)dF(s)\right]+ \\
&\text{(II)} \qquad + \phi_d\iota\,(1-F(d))\,(v(m,d)+u)+ \\
&\text{(II)} \qquad + \phi_d\iota\int_m^d (v(m,d-s)+\kappa_d)\,dF(s)+ \\
&\text{(II)} \qquad + \phi_d\iota\int_0^m \min\left\{v(m-s,d), v(m,d-s)+\kappa_d\right\}dF(s)+ \\
&\text{(III)} \qquad + (1-\phi)\phi_d(1-\iota)\,(1-F(d))\left(v(m,0)+\kappa_d+\widetilde{b}_d\right)+ \\
&\text{(III)} \qquad + (1-\phi)\phi_d(1-\iota)\int_m^d (v(m,d-s)+\kappa_d)\,dF(s)+ \\
&\text{(III)} \qquad + (1-\phi)\phi_d(1-\iota)\int_0^m \min\left\{v(m-s,d), v(m,d-s)+\kappa_d\right\}dF(s)+ \\
&\text{(IV)} \qquad + \phi(1-\phi_d)(1-\iota)\,(1-F(m))\,(v(m,d)+\kappa)+ \\
&\text{(IV)} \qquad + \phi(1-\phi_d)(1-\iota)\int_0^m \min\left\{v(m-s,d)\ ,\ v(m,d)+\kappa\right\}dF(s)+ \\
&\text{(V)} \qquad + \phi\phi_d(1-\iota)\,(1-F(d))\min\left\{v(m,d)+\kappa\ ,\ v(m,0)+\kappa_d+\widetilde{b}_d\right\}+ \\
&\text{(V)} \qquad + \phi\phi_d(1-\iota)\int_m^d \min\left\{v(m,d)+\kappa\ ,\ v(m,d-s)+\kappa_d\right\}dF(s)+ \\
&\text{(V)} \qquad + \phi\phi_d(1-\iota)\int_0^m \min\left\{v(m-s,d)\ ,\ v(m,d)+\kappa\ ,\ v(m,d-s)+\kappa_d\right\}dF(s).
\end{aligned}
$$

B.2 Additional figures

Figure B1: Adjustment policy $\mathcal{A}(m,d)$

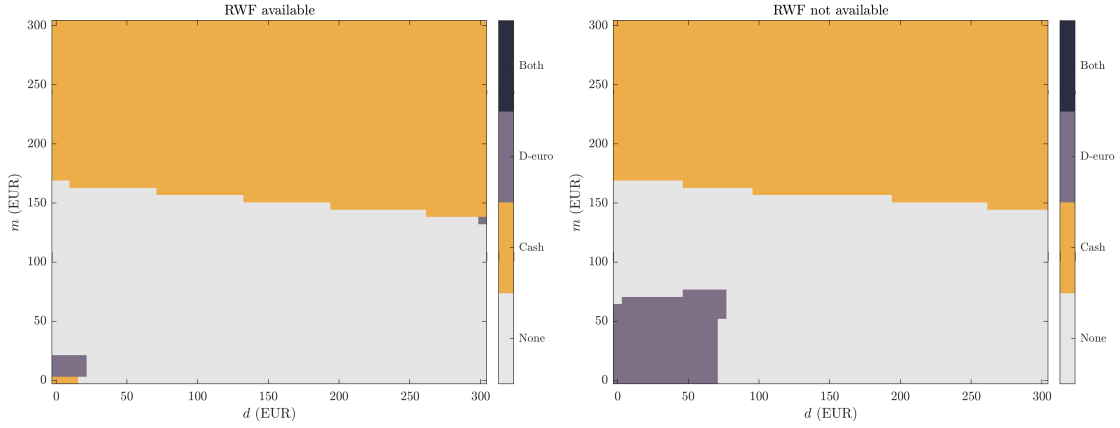
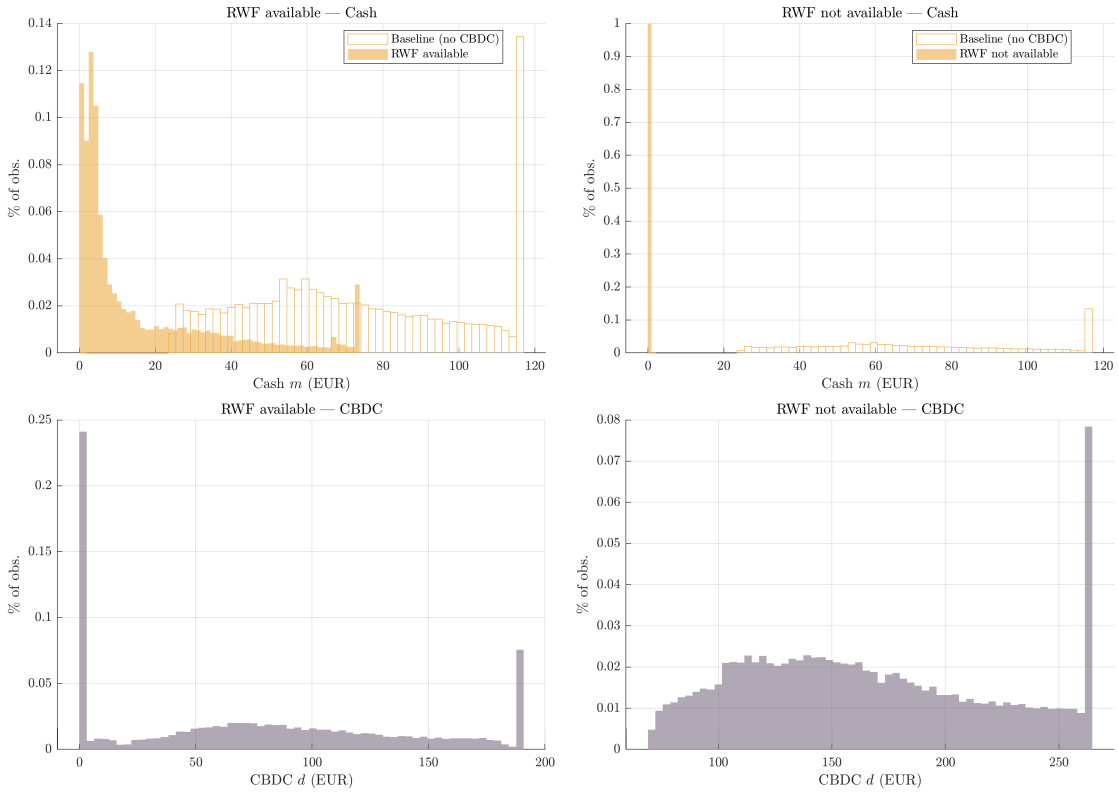
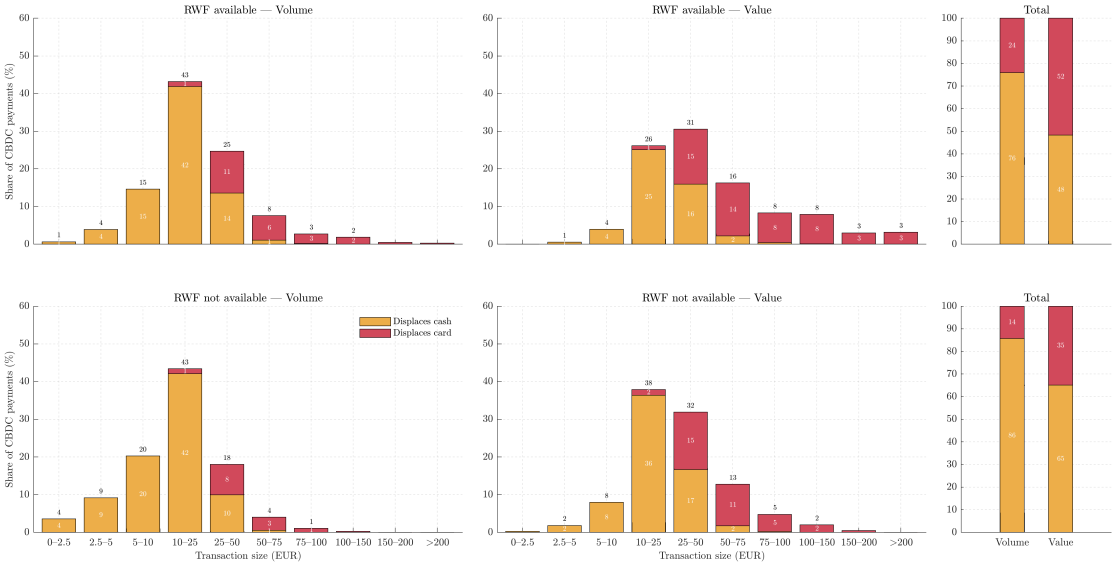


Figure B2: Stationary distributions for cash and CBDC holdings.



Note: The Figure displays the stationary distributions h_m and h_d for our baseline calibration (see Section 3 for details) under both the case in which the RWF is available and free to use and the case in which it is not.

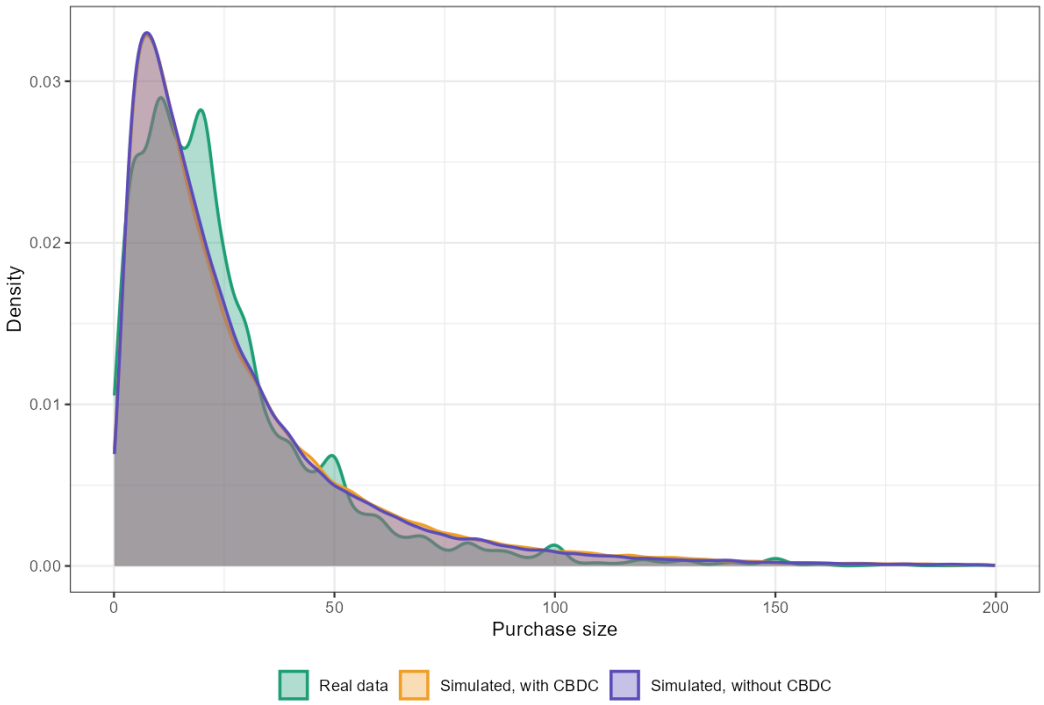
Figure B3: Cash and card substitution patterns for different transaction sizes s



Note: Model-implied patterns of substitution between cash, cards and CBDC when settling transactions of increasing size s. Transactions are grouped into 10 bins (same number of purchases in each bin). For each bin, we compute the share of cash and card transactions in the pre-CBDC world, and then we visualize the resulting decomposition of the CBDC share. The right panel of each plot summarizes substitution patterns, both in value and in volume terms, displaying how much CBDC transactions are displacing cash and card payments.

C Transaction size distribution fit

Figure C1: Distribution of transaction size



Note: Density estimation of transaction size distribution in SPACE 2024 data (green) and in the simulated data from our model, with and without CBDC (yellow and purple).